Seeking Practical CDCL Insights from Theoretical SAT Benchmarks

Jakob Nordström

KTH Royal Institute of Technology
Stockholm, Sweden

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Joint work with Jan Elffers, Jesús Giráldez Cru, Stephan Gocht, and Laurent Simon

The Satisfiability Problem (SAT)

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- $\wedge$ means all constraints should hold simultaneously
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Surprisingly rich formalism for expressing many real-world applied problems

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Want a **deeper understanding** of how these solvers actually work

How to Analyse Behaviour of CDCL Solvers?

Applied approach

- Instrument solver with combinations of heuristics
- Run on SAT competition benchmarks
- Few and heterogeneous benchmarks
  - Poorly understood properties
  - Isolated data points
- Some work in [LM02, KSM11, SM11] — hard to draw conclusions

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Theoretical approach

- CDCL solvers search for proofs in resolution proof system
- Relate CDCL performance to proof complexity measures?
- Only considers existence of proofs, not algorithmic search
- Only asymptotic results (sometimes for gigantic formulas)
- Papers [JMNŽ12, MN14] generated more questions than answers...

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Our combined approach

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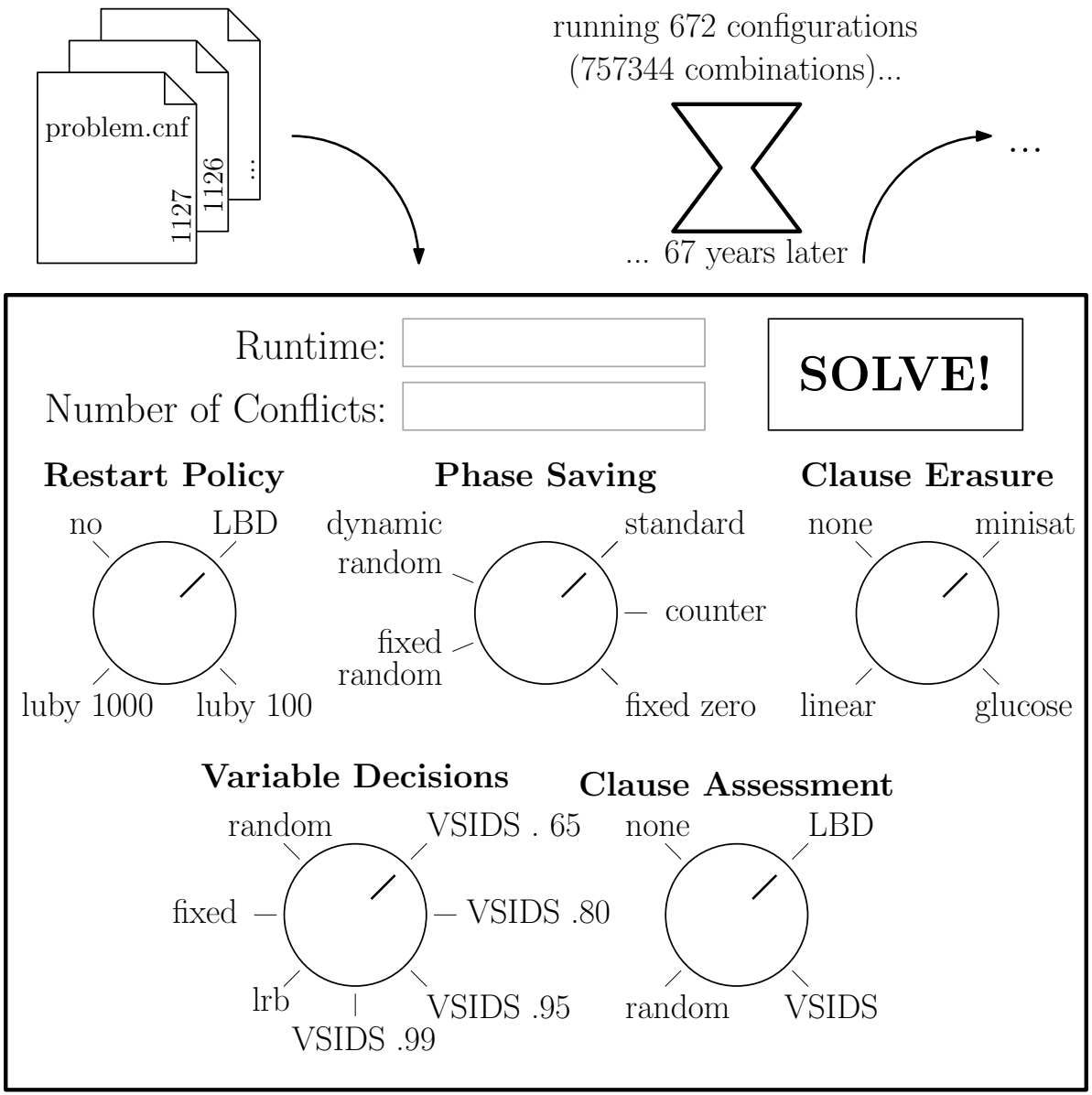
- Extremal benchmarks w.r.t. different properties
  $\Rightarrow$ Different benchmarks will "stress-test" different heuristics
- Code up instrumented solver with wide selection of heuristics
  - More complicated than it sounds — some heuristics tightly integrated
  - Run on (essentially full) cross product of heuristics
  - Which heuristics are important when?
  - How do heuristics interact?

Our Set-up


problem.cnf
1127
1126
...
running 672 configurations
(757344 combinations)...
... 67 years later
...
Runtime:
Number of Conflicts:
SOLVE!
Restart Policy
no
LBD
luby 1000
luby 100
Phase Saving
dynamic
standard
random
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fixed zero
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none
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- 672 solver configurations
- More than 500,000 hours (67 years) of computations

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- How to make sure results are significant?
- Solvers deterministic — standard statistic tools don't seem to apply

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Rest of this talk: some example findings

- Restarts
- Memory management
- Branching heuristic

Restarts

- Fast restarts crucial for CDCL solvers in practice
- Also appear in proofs that CDCL searches efficiently (kind of) in resolution [AFT11, PD11]
- But do restarts increase theoretical reasoning power? Open...
- Run experiments on benchmarks where "full power of resolution" needed [AJPU07] to gather "circumstantial evidence"?

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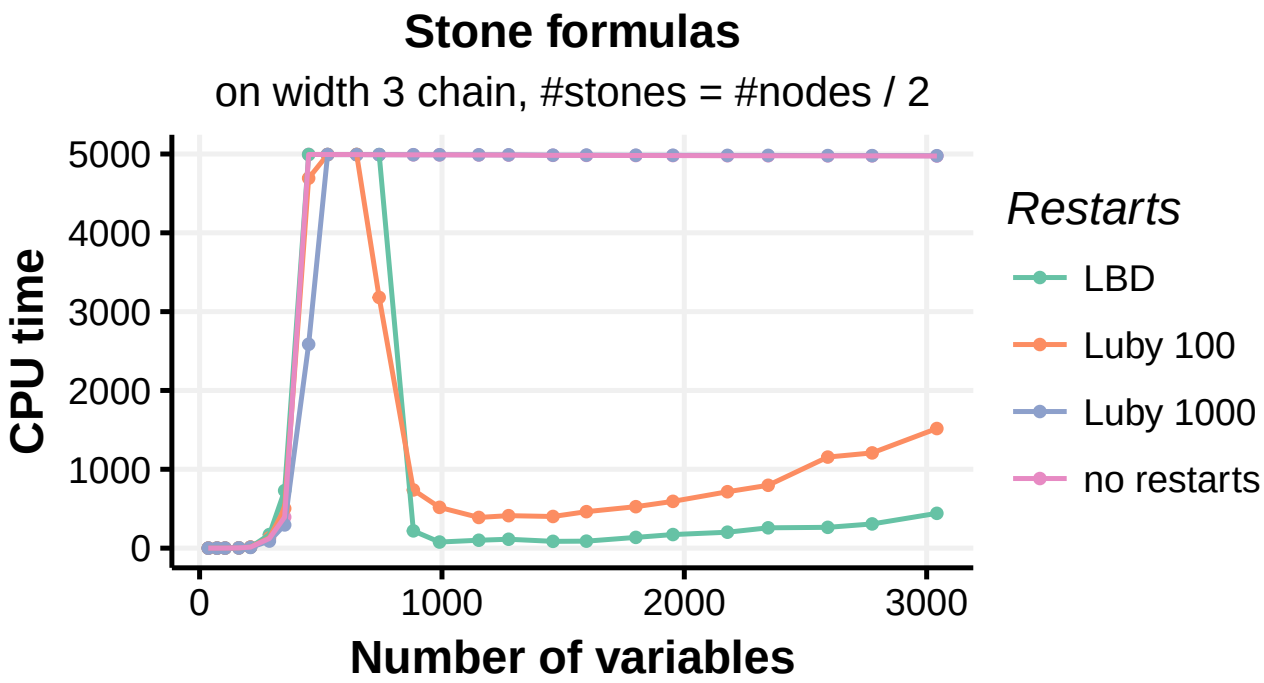

Memory Management: Time-Space Trade-offs

- CDCL solvers very aggressively minimize memory usage
- Dramatic time-space trade-offs in theory [BN11, BBI12, BNT13]
- Could this happen in practice?

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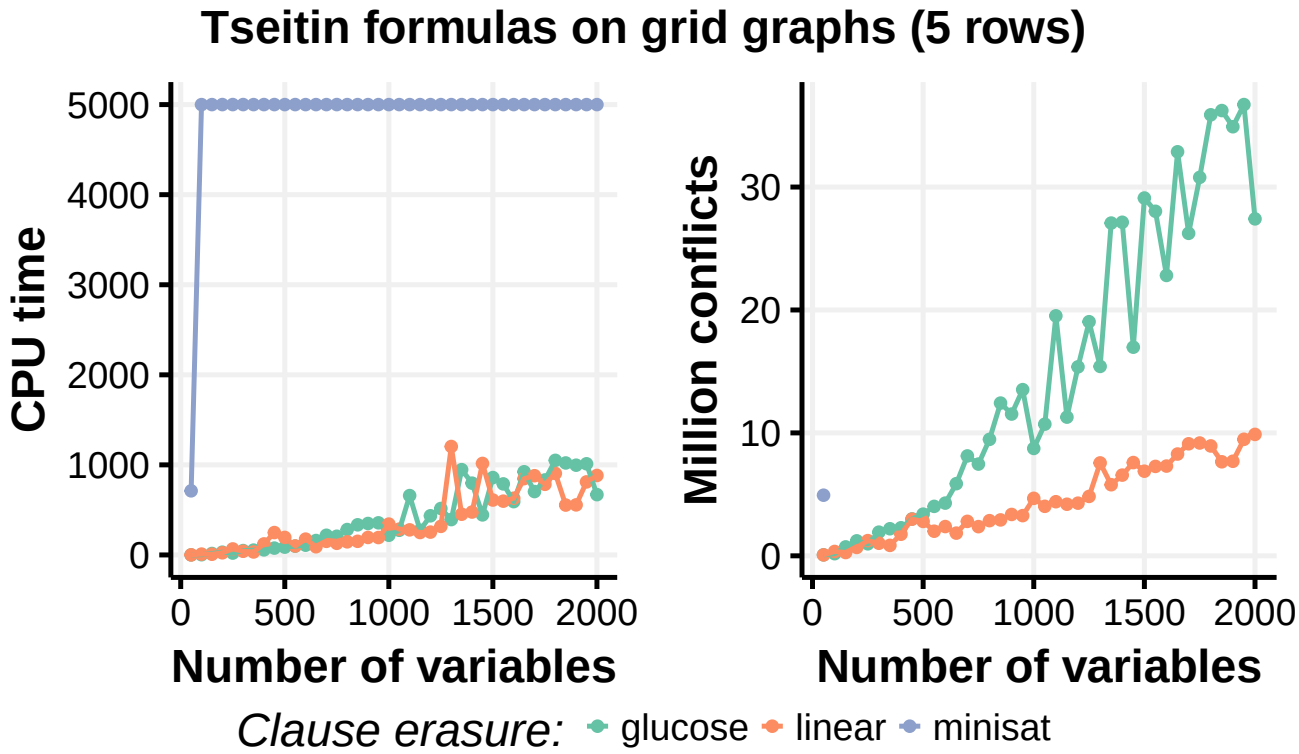

Database size: minisat $\left(\sim N^{0.25}\right)$ < glucose $\left(\sim N^{0.5}\right)$ < linear $\left(\sim N\right)$ [$N = \#\text{conflicts}$]

Memory Management: Clause Assessment

- When time to purge database, how to identify useful clauses to keep?
  - Activity-based [ES04]
  - Literal block distance (LBD) [AS09]
  - Random (control)

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- When time to purge database, how to identify useful clauses to keep?
  - Activity-based [ES04]
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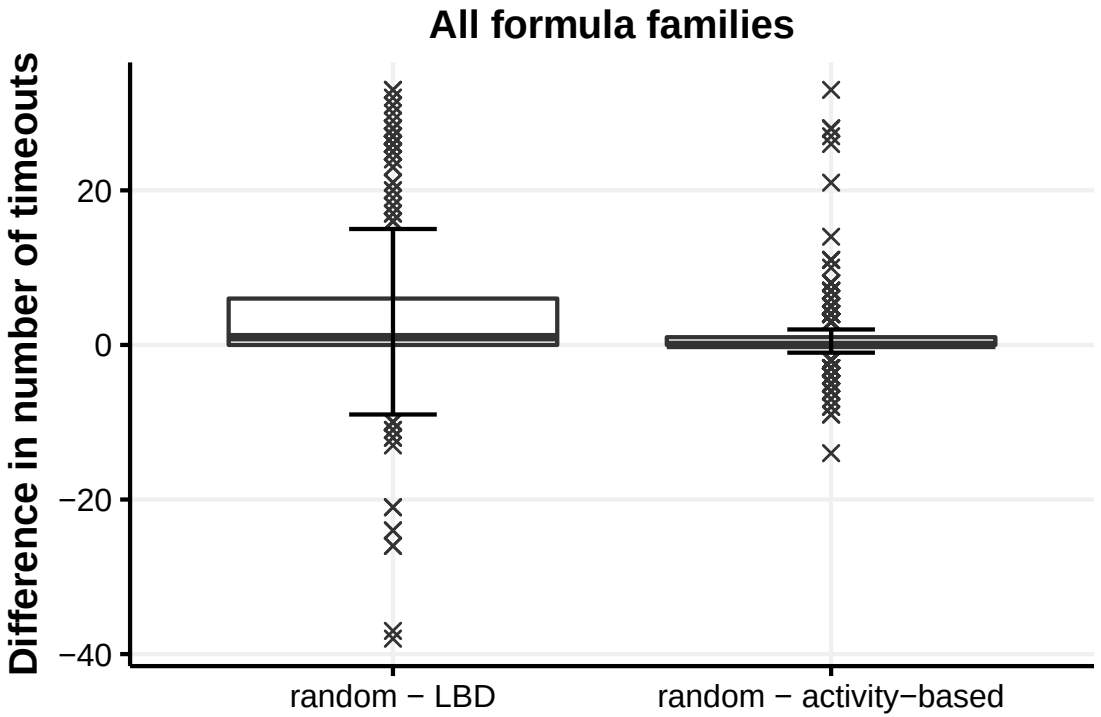
  - Random (control)
- LBD quite successful, especially when space is getting tight
- Activity-based indistinguishable from random (& random OK-ish)



Branching Heuristics: Importance of Memory Horizon

- Branch on variable appearing most often in recent conflicts
  + exponential decay to reward recent conflicts: VSIDS [MMZ$^+$01]
    - Low VSIDS factor = short memory
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- Choice high/low depends on instance (but random choice always bad)

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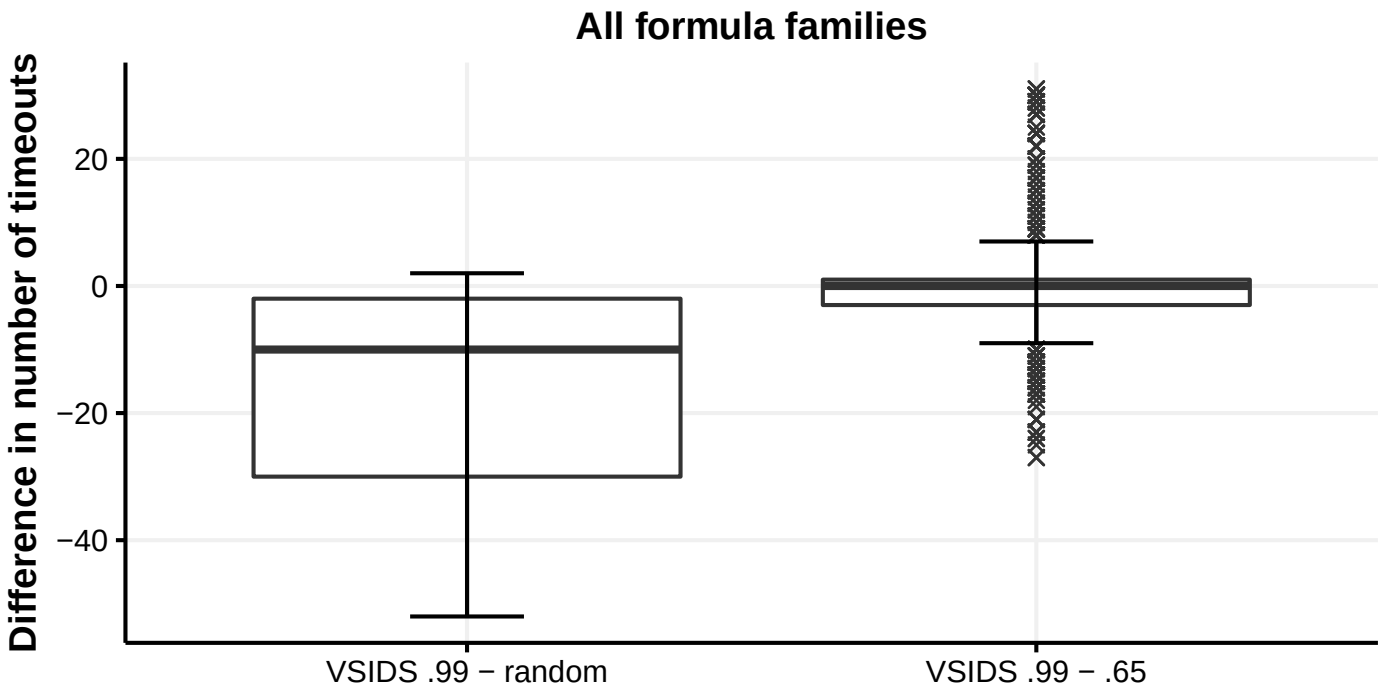
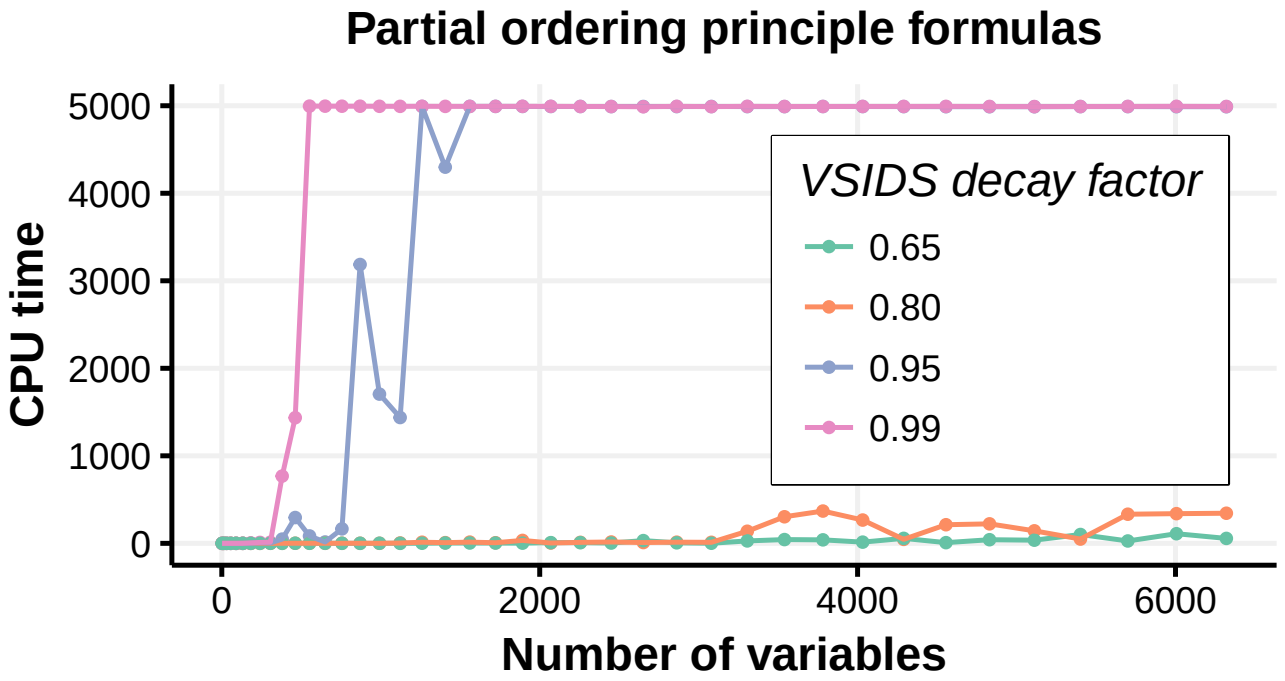


Branching Heuristics: A Particularly Dramatic Example



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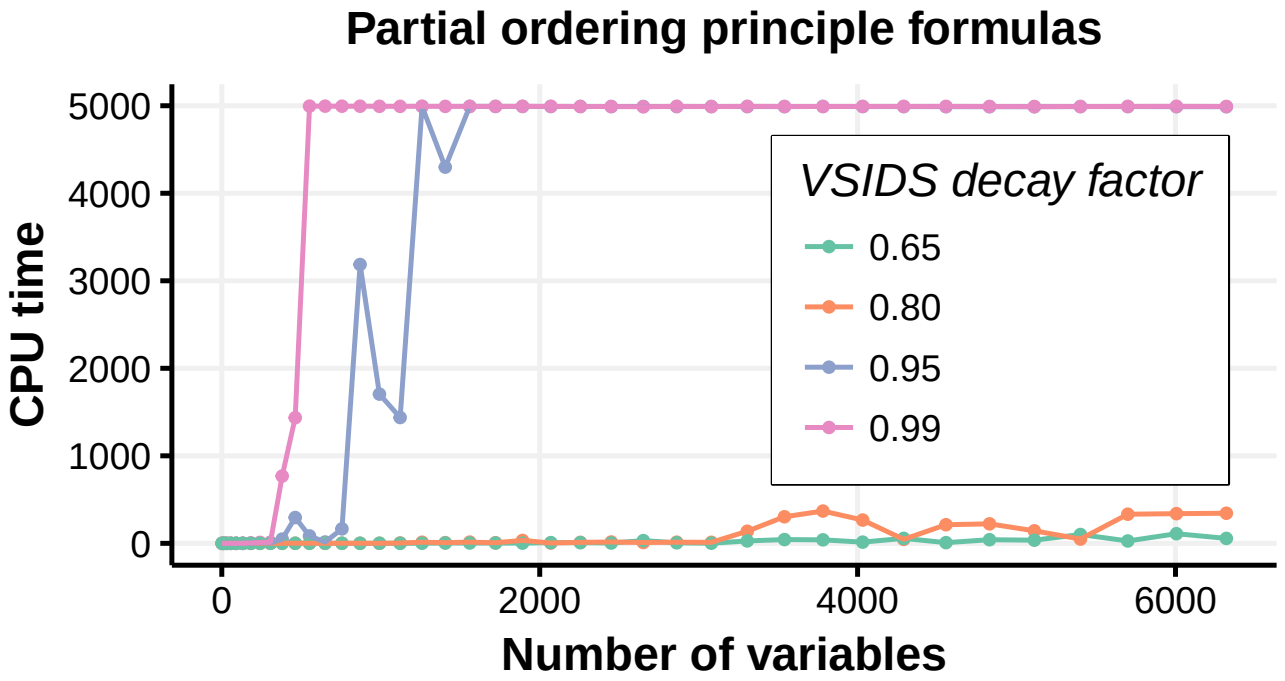

- What's going on? We're not sure...
- Instances very hard for tree-like resolution ($\approx$ DPLL) [BG01, BW01]
- Low VSIDS factor $\Rightarrow$ search focus locally on latest conflicts
- Maybe high VSIDS factor $\Rightarrow$ closer to DPLL-style search?!
  (Consistent with our experiments, but more data & insights needed)

Take-Away Message

This work

Goal: Not better solvers, but **understand** how best solvers work

Approach: Harness theory results for empirical study of heuristics

Conclusion: Yes, can get practical insights from theory benchmarks!

- Sometimes confirms conventional wisdom — nice to get evidence
- Sometimes more surprising results — raise questions for further study

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Directions for future research

More in-depth study of intriguing questions raised

- Restarts: Only frequency important or also exact timing?
- Memory management: Trade-off speed/quality of proof search
- Branching: VSIDS factor crucial — how to get it right?
- Phase saving: Super-important also for unsatisfiable instances — why?

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Analogous study on industrial benchmarks

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Thank you for your attention!

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